PROJECT TEAM
SERENA

E-mail: serena@inria.fr
Inria Paris
Adress: 2, rue Simone Iff
75012 PARIS

GENFIELD
A parallel software for the generation of stationary Gaussian random fields

https://gitlab.inria.fr/slegrand/Genfield_dev

PICHOT Géraldine Inria Paris
LEGRAND Simon Inria Paris
ERHEL Jocelyne Inria Rennes
OUMOUNI Mestapha Nantes University

Introduction

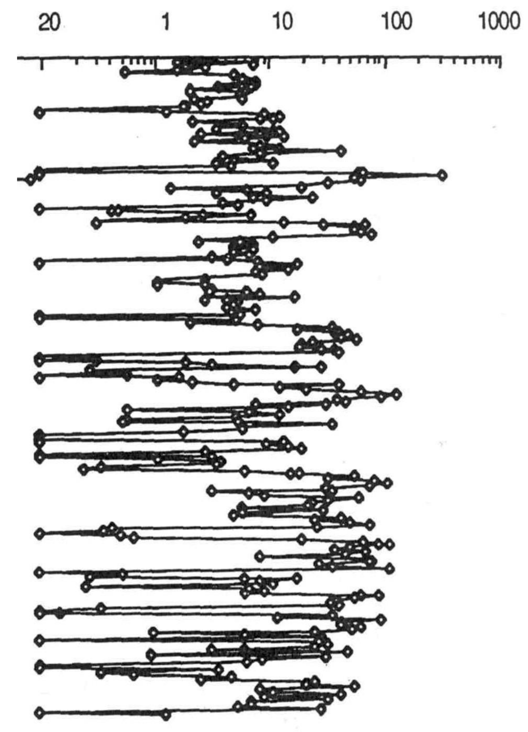
- **Context**

In hydro-geology, natural permeability fields are classically modeled by a **second-order stationary field** which has a lognormal distribution, with an isotropic exponential covariance function. In this poster, we expose an algorithm to generate Gaussian random fields based on the **Circulant Embedding method** and its implementation in **C++/MPI**.

- **Useful in different applications**

- sustainable management of groundwater resources
- pollution propagation studies
- feasibility studies of waste storage in deep geological media

Sand and gravel deposits in Switzerland, Gelhar [1993]



permeability (md)

General algorithm in 1D

Let $\Omega = [0, 1]$. We discretize Ω over a regular grid composed of $N + 1$ elements.

To generate a realization of the random vector $\boldsymbol{Y}$ of normal variables with zero mean and autocovariance matrix $\boldsymbol{R}$, we use the following algorithm:

1. Factorize $\boldsymbol{R} = \boldsymbol{B}\boldsymbol{B}^T$
2. Generate a vector $\boldsymbol{\theta} = (\theta_0, \ \dots \ , \ \theta_N)^T$ as a realization of uncorrelated random normal variables with zero mean.
3. One realization is obtained by $\boldsymbol{Y} = \boldsymbol{B}\boldsymbol{\theta}$.

The usual method to decompose $\boldsymbol{R}$ is Cholesky. Its cost is in $O(N^3)$ and becomes too prohibitive for large problems.

The **circulant embedding method** is a very interesting alternative as it relies on the Discrete Fourier Transform. This latter has an efficient $\boldsymbol{O(N\log N)}$ implementation in the **FFTW library**

Circulant embedding method in 1D

Algorithm when no padding is required (i.e. when all eigenvalues are positive in step 2.):

1. Sample the autocovariance function to get circulant vector $\boldsymbol{a} \in \mathbb{R}^{2N}$.
2. Compute the vector of eigenvalues $\boldsymbol{s} = \boldsymbol{F}\boldsymbol{a}$ with FFT and set $\boldsymbol{D} = Diag(\boldsymbol{s})$.
3. Generate two realizations of standard normal random vectors of size $2N, \boldsymbol{\theta}^{re}$ and $\boldsymbol{\theta}^{im}$:
$$\boldsymbol{\theta} = \boldsymbol{\theta}^{re} + i\boldsymbol{\theta}^{im},$$
4. Apply iFFT to compute $\boldsymbol{C}^*\boldsymbol{\theta}$ with $\boldsymbol{C}^* = \frac{1}{\sqrt{2N}}\boldsymbol{F}^*\boldsymbol{D}^{\frac{1}{2}}$
5. Take
$$\boldsymbol{Y_1} = (\boldsymbol{C}^*\boldsymbol{\theta})^{re}(0 : N) \text{ and } \boldsymbol{Y_2} = (\boldsymbol{C}^*\boldsymbol{\theta})^{im}(0 : N)$$

Our objective is to introduce a new parallel implementation of the circulant embedding method.

C++/MPI Implementation

0. Parameters parsing and padding estimation — Boost

↓

1. Covariance function discretization — MPI

↓

2. Fast Fourier Transform — MPI, FFTW3

↓

3. Gaussian random vectors — Boost, Rngstream

↓

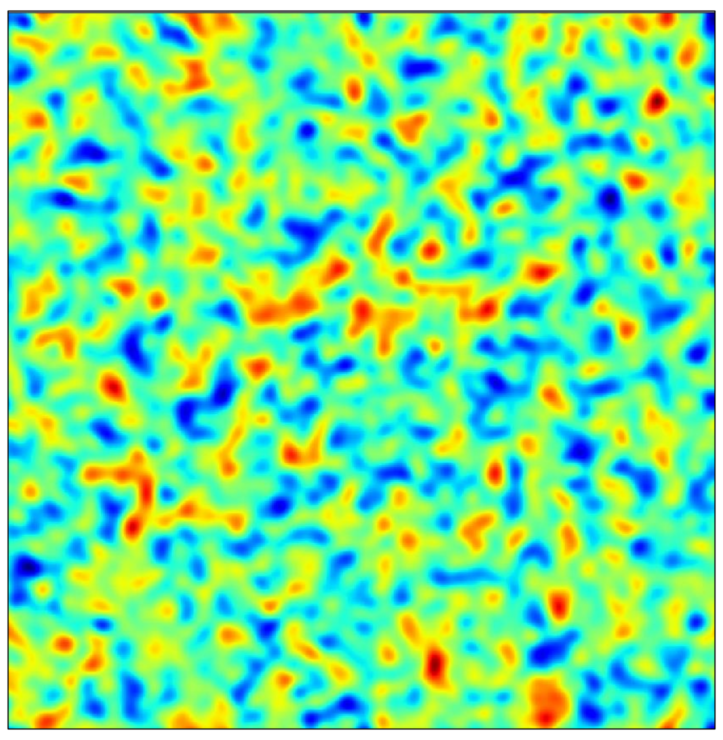
4. Inverse Fast Fourier Transform — MPI, FFTW3

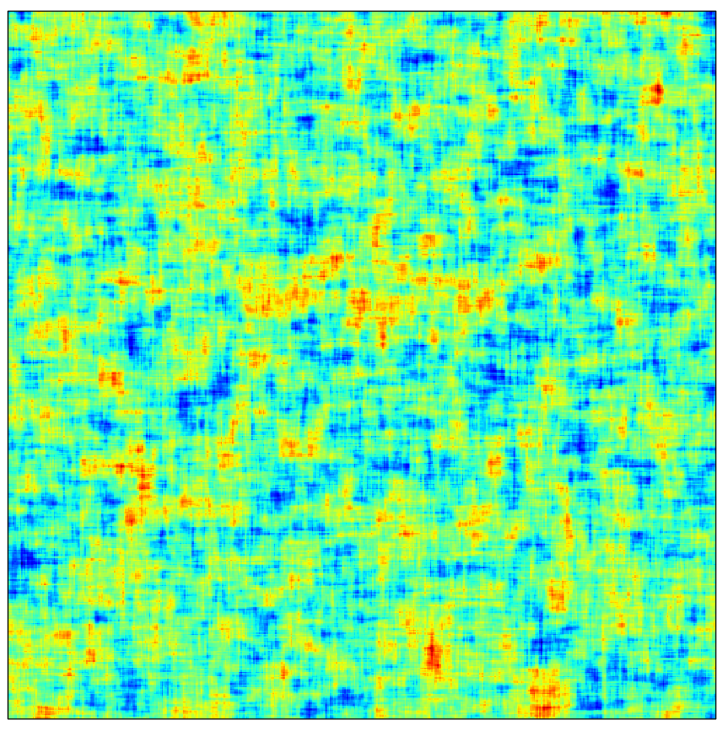
↓

5. Field retrieval — MPI

Parallel sections   Sequential sections

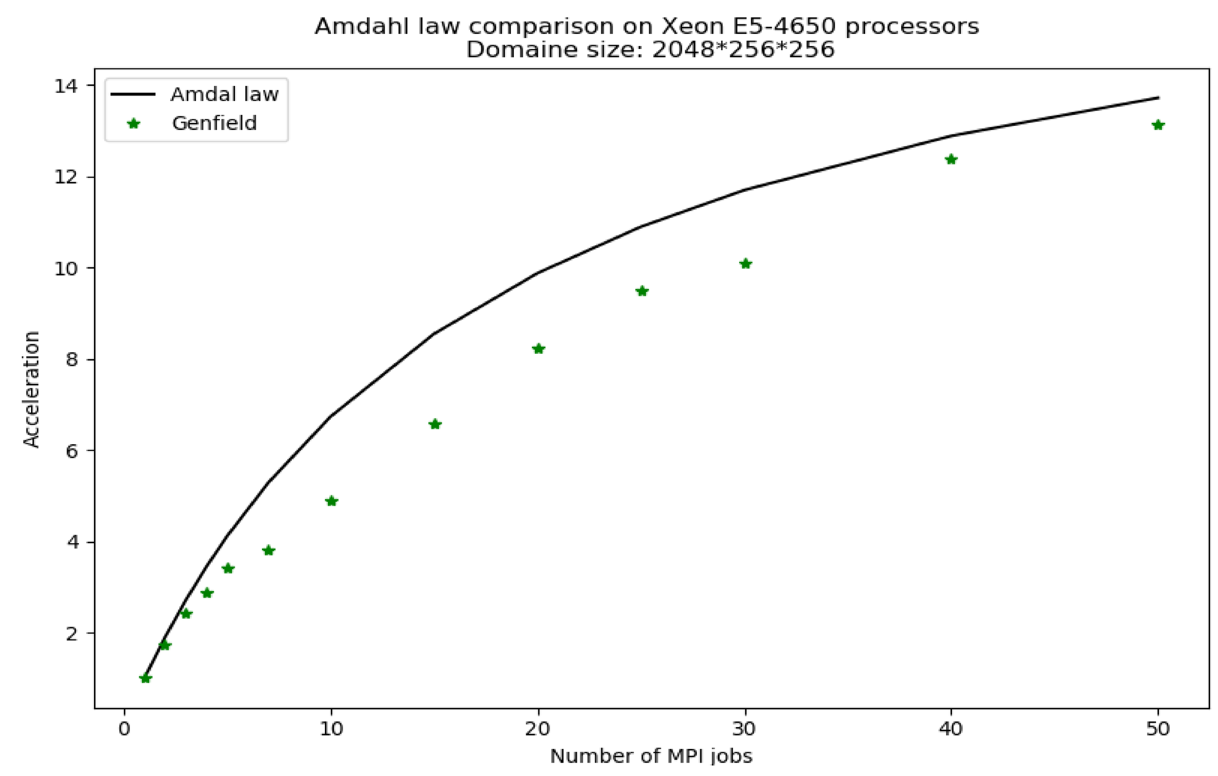
Examples of fields in 2D

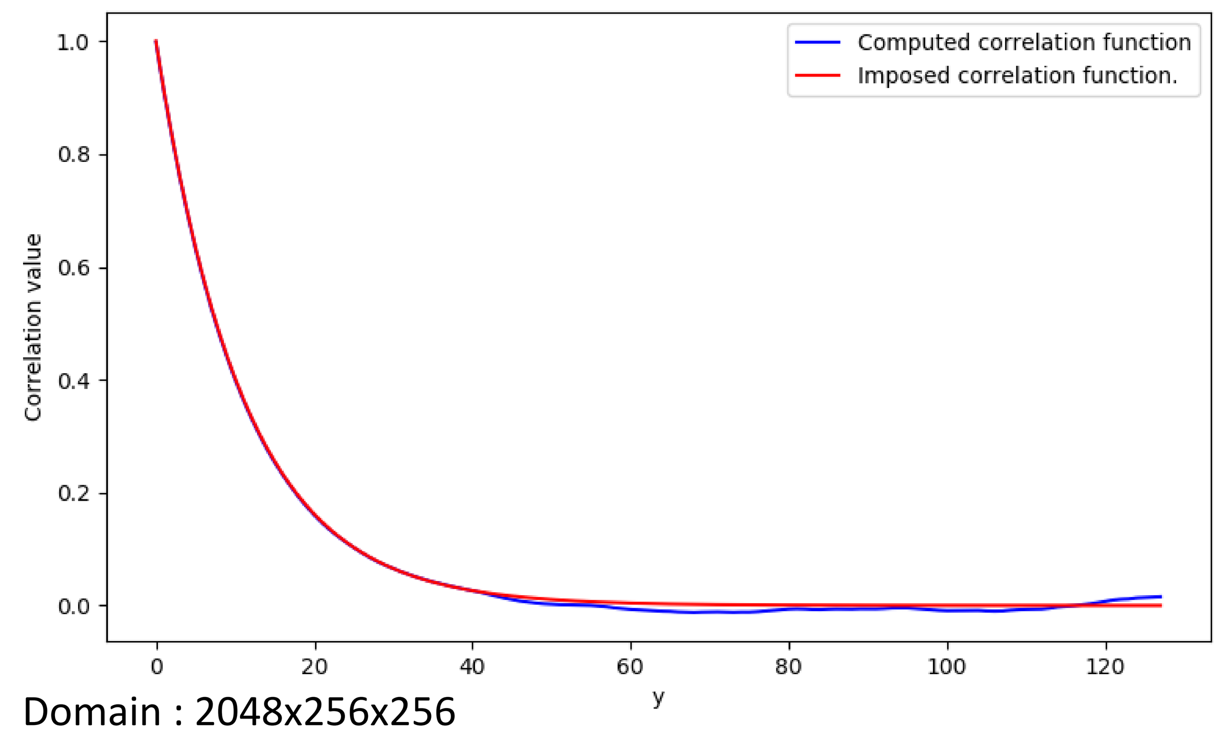




Domain : 256 x 256
Up: Gaussian covariance function
Bottom: Exponential covariance function

Performances in 3D





Domain : 2048x256x256
Up: Speedup curve compared with Amdahl law (5% sequential)
Bottom: comparison between the computed and the imposed covariance function

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Acknowledgments. Computer simulations were performed on the ADA cluster at the **Institute for Development and Resources in Intensive Scientific Computing**, France.

Perspectives

- Parallelize the last 5% of the code, i.e. the generation of the gaussian random vectors using the pseudo-random numbers generator Rngstream written by P. L'Ecuyer.
- Extend the inputs/outputs to other formats such as xml, json and hdf5.

École des Ponts ParisTech
UNIVERSITÉ DE NANTES